Magnetic instability of a two-dimensional Anderson non-Fermi Liquid

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We show that in the Anderson model for a two-dimensional non-Fermi liquid a magnetic instability can lead to the itinerant electron ferromagnetism. The critical temperature and the susceptibility of the paramagnetic phase have been analytically calculated. The usual Fermi behaviour is re-obtained taking the anomalous exponent to be zero.

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Since the discovery of high-T_c superconductors there has been a great interest in the study of the non-Fermi liquid model in two dimensions (2D). The model proposed by Anderson[1,2] has a phenomenological character because the form of the Green's function has been taken from the one-dimensional Luttinger liquid. Later the model has been used for both the normal and superconducting state[3–8], and the usual BCS critical temperature was re-obtained for the case of a zero anomalous exponent on the Green function.

However, the electron-hole channel and the possibility for the occurrence of the itinerant electron ferromagnetism was not studied for this model. A recent calculation[9] of the magnetic susceptibility of a 1D system, done with the renormalization group method and the Monte Carlo simulation, showed that the magnetic susceptibility $\chi(T)$ becomes constant as $T \to 0$, but with an infinite slope, in contrast with the normal Fermi liquid.

In the present paper we shall assume the existence of a 2D non-Fermi liquid described by the Green function:

$$G(\mathbf{k},\omega) = \frac{g(\alpha)e^{i\phi}}{\omega_c^{\alpha}\left[\omega - \varepsilon(\mathbf{k}) + i\delta\right]^{1-\alpha}} \tag{1}$$

where α is the anomalous exponent, $\phi = -\pi\alpha/2$ is a phase factor introduced in order to respect the time reversal symmetry of the Green's function, $\varepsilon(\mathbf{k})$ is the kinetic energy of the electrons and $g(\alpha) = \pi\alpha/2\sin(\pi\alpha/2)$ is a factor introduced to preserve the equal time anticomutation relations of the electrons. The form of the Green's function is assumed to be given by Eq. (1) as long as $-\omega_c \le \omega \le \omega_c$, ω_c being the energy cutoff specific for the model. The exponent α is non-universal[3,4] and is considered to has a value $0 < \alpha < 1/2$[1,2,4,7,8].

In order to study the electron-hole instability we calculate the polarisation $\Pi(\mathbf{q},\omega$ defined by

$$\Pi(\mathbf{q},\omega_m) = e^{-i\pi\alpha}\frac{g^2(\alpha)}{\omega_c^{2\alpha}}\int\frac{d^2\mathbf{k}}{(2\pi)^2}S(\mathbf{k},\mathbf{q},i\omega_m) \tag{2}$$

where

$$S(\mathbf{k},\mathbf{q},i\omega_m) = T\sum_n \frac{1}{[i\omega_n - \xi(\mathbf{k})]^{1-\alpha}} \times \frac{1}{[i\omega_n - i\omega_m - \xi(\mathbf{k}-\mathbf{q})]^{1-\alpha}} \tag{3}$$

with $\xi(\mathbf{k}) = k^2/2m - \mu + nV/2$, μ being the chemical potential and $nV/2$ the Hartree energy of the n electrons. The summation over the Matsubara frequencies can be substituted by an complex integral and we obtained for the polarisation:

$$\Pi(\mathbf{q},i\omega_m) = 2e^{-i\pi\alpha}\frac{\sin[\pi(1-\alpha)]}{\pi}\frac{g^2(\alpha)}{\omega_c^{2\alpha}}\int\frac{d^2\mathbf{k}}{(2\pi)^2} \times \int_{\xi(\mathbf{k})}^{\infty} dx \frac{n_F(x)}{[x-\xi(\mathbf{k})]^{1-\alpha}[x-\xi(\mathbf{k}-\mathbf{q})-i\omega_m]^{1-\alpha}} \tag{4}$$

Following the same way as in Ref. 11 we calculate the polarisation as

$$\begin{aligned}\Pi(\mathbf{q},i\omega_m) = {} & 2N(0)e^{-i\pi\alpha}\frac{g^2(\alpha)}{\omega_c^{2\alpha}} \\ & \times \frac{\sin[\pi(1-\alpha)]}{\pi}\left[\Gamma(2\alpha-1)S_1(\beta,\tilde{\mu})\right. \\ & - i\omega_m(\alpha-1)\Gamma(2\alpha-2)S_2(\beta,\tilde{\mu}) \\ & \left. - \frac{v_F^2q^2}{2}(\alpha-1)S_3(\beta,\tilde{\mu})\right]\end{aligned} \tag{5}$$

where

$$S_1(\beta,\tilde{\mu}) = \sum_{m=0}^{\infty}(-1)^m\frac{e^{\beta\tilde{\mu}(m+1)}}{[\beta(m+1)]^{2\alpha}} \tag{6}$$

$$S_2(\beta,\tilde{\mu}) = \sum_{m=0}^{\infty}(-1)^m\frac{e^{\beta\tilde{\mu}(m+1)}}{[\beta(m+1)]^{2\alpha-1}} \tag{7}$$

$$S_3(\beta,\tilde{\mu}) = \sum_{m=0}^{\infty}(-1)^m\frac{e^{\beta\tilde{\mu}(m+1)}}{[\beta(m+1)]^{2\alpha-2}} \tag{8}$$

with $\tilde{\mu} = \mu - nV/2$.

The paramagnetic susceptibility given by

$$\chi(T) = -2\mu_0^2 Re\Pi(0,0) \tag{9}$$

has been calculated as

$$\chi(T) = -2\mu_0^2 N(0)\frac{g^2(\alpha)}{\omega_c^{2\alpha}}\frac{2^{2\alpha}}{\sqrt{\pi}}\cos(\pi\alpha)\frac{\Gamma(\alpha-1/2)}{\Gamma(1-\alpha)}S_1(\beta,\tilde{\mu}) \tag{10}$$

where N(0) is the density of states. In the limit $\alpha \to 0$ we calculated from Eq. (6)

$$S_1^0(\beta,\mu) = f_{F-D}(nV/2-\mu) \tag{11}$$

where $f_{F-D}(x)$ is the Fermi-Dirac distribution function, and using this result the paramagnetic susceptibility at $T \to 0$ becomes

$$\chi_p(T\to 0) = 2\mu_0^2 N(0) \tag{12}$$

a result which is in fact the Pauli paramagnetic susceptibility.

The transition temperature can be calculated by considering also the equation for the particle density in order to eliminate the effective chemical potential $\tilde{\mu}$. The general equation for the electron density is

$$n = T\sum_n \int \frac{d^2\mathbf{k}}{(2\pi)^2} G(\mathbf{k}, i\omega_n) \tag{13}$$

where $G(\mathbf{k}, i\omega_n)$ is given by Eq. (1). Performing the summation over the Matsubara frequencies ω_n and the integral for a constant density of states we obtain

$$n(T,\tilde{\mu}) = \frac{N(0)g(\alpha)}{\omega_c^\alpha}\Gamma(\alpha)\frac{\sin(\pi\alpha/2)}{\pi}S_0(\beta,\tilde{\mu}) \tag{14}$$

If we consider $S_0(\beta,\tilde{\mu})$ well approximated by its value at $\alpha = 0$ we have

$$S_0(\beta,\tilde{\mu}) \cong \frac{1}{\beta}\ln\left[1+e^{\beta\tilde{\mu}}\right] \tag{15}$$

and we get

$$e^{\beta\tilde{\mu}} \cong e^{\beta n/B(\alpha)} - 1 \tag{16}$$

where

$$B(\alpha) = \frac{N(0)g(\alpha)}{\omega_c^\alpha}\Gamma(\alpha)\frac{\sin(\pi\alpha/2)}{\pi}$$

In order to calculate the critical temperature T_c bellow the itinerant ferromagnetism appear we will use the relation

$$1 + VRe\Pi(\mathbf{q}, i\omega_n) = 0 \tag{17}$$

in the limit $q = 0$ and $i\omega_n = 0$. We mention that according to the Mermin-Wagner theorem the ferromagnetic phase is destroyed by the fluctuations and T_c is in fact zero. However, the weak three dimensionality which is present in High-T_c materials gives us the possibility to use Eq. (17) for the calculation of the critical temperature. We can introduce this effect phenomenologically taking instead of Eq. (17) the equation $1 + VRe\Pi(0,0) = 0.01$ which corresponds to 10^2 times enhancement of paramagnetic susceptibility. The qualitative phase diagram is not affected by these details. We calculated from Eq. (4) the real part of $\Pi(\mathbf{q}, i\omega_n)$ as

$$Re\Pi(0,0) = 2N(0)\Gamma(2\alpha-1)\frac{g^2(\alpha)}{\omega_c^{2\alpha}}\frac{\sin\pi(1-\alpha)}{\pi}S_1(\beta,\tilde{\mu}) \tag{18}$$

If we consider again that $S_1(\beta,\tilde{\mu})$ is well approximated by its value for $\alpha \to 0$ we have

$$S_1(\beta,\tilde{\mu} = \frac{e^{\beta\tilde{\mu}}}{e^{\beta\tilde{\mu}}+1} \tag{19}$$

which together with Eq. (18) gives for the real part of the polarisation the value

$$Re\Pi(0,0) = A(\alpha)\frac{e^{\beta\tilde{\mu}}}{e^{\beta\tilde{\mu}}+1} \tag{20}$$

with

$$A(\alpha) = 2\Gamma(2\alpha-1)\frac{g^2(\alpha)}{\omega_c^{2\alpha}}N(0)\frac{\sin\pi\alpha}{\pi}$$

Using this result we calculate the critical temperature as

$$T_c(\alpha) = -\frac{n}{B(\alpha)}\frac{1}{\ln\left[1-\frac{1}{V|A(\alpha)|}\right]} \tag{21}$$

From Eq.(21) we get a critical condition for the existence of the ferromagnetic order expressed as

$$V_c < \frac{1}{|A(\alpha)|} \tag{22}$$

In the limit $\alpha \to 0$ the critical temperature becomes identical with the one obtained for a three dimensional itinerant electron ferromagnet. This result is given by the fact that in both cases the integral over the energy variable is performed at the Fermi surface using a constant density of states. For a 2D electron system a more realistic description will be the one in terms of a van Hove density of state. We expect as for the superconducting critical temperature calculated in Ref. 8 that the ferromagnetic critical temperature will be enhanced by the energy dependence of the density of states. Anyway such a calculation is much more difficult and the results will be published in another work.

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